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Fixed-Trim Re-Entry Guidance Analysis

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The terminal guidance problem for a fixed-trim re-entry body is formulated with the objective of synthesizing a closed-loop steering law. A transformation of variables and subsequent linearization of the motion, with the sight-line to the target as a reference, reduces the order of the state system for the guidance problem. The reduced-order system, although nonlinear and time-varying, is simple enough to lend itself to synthesis of a class of guidance laws. A generalization of the feedforward device of classical control theory is successfully employed for compensation of roll autopilot lags. The proposed steering law exhibits superior miss-distance performance in a computational comparison with existing fixed-trim guidance laws.

Nomenclature

A	= magnitude of the acceleration of the vehicle
C_D	= drag coefficient of the vehicle
C_{L_α}	= lift-curve slope for the vehicle
D	= drag force on the vehicle
g	= acceleration due to gravity
k	= guidance constant or gain
$\hat{k}$	= time-varying guidance gain
L	= lift force on the vehicle
m	= mass of the vehicle
p_c	= commanded roll rate
p_L	= roll rate limit of the vehicle
q	= dynamic pressure
r	= magnitude of the position of the vehicle with respect to the target
S	= reference area of the vehicle
t	= elapsed time of flight
t_f	= final time of flight
T	= generic coordinate transformation matrix
V	= airspeed of the vehicle
$(x,y,z)^T$	= position coordinates of the vehicle with respect to a target-centered, inertial coordinate system
y	= vehicle navigation system output vector
α_T	= total, trim angle of attack of the vehicle
γ,χ	= flight path and heading angles, respectively, of the vehicle with respect to the nonrolling velocity fixed frame
δ	= polar orientation of the projection of the sight line to the aimpoint with respect to the nonrolling velocity frame
ϵ	= angle between the velocity and sight-line vectors
ρ	= magnitude of the sight-line vector
ρ_a	= air density
$(\rho_{v_x},\rho_{v_y},\rho_{v_z})^T$	= line-of-sight vector, presented in the nonrolling velocity fixed frame
ρ_z	= component of the sight line perpendicular to the velocity vector
τ_ϕ	= roll-autopilot/airframe time constant
ϕ_v	= roll orientation of the lift vector with respect to the nonrolling velocity fixed frame
ϕ_c	= commanded value of ϕ_v
ϕ_e	= roll angle error

Subscripts

0	= constant, reference, or initial value
x,y,z	= components along the inertial axes
x_v,y_v,z_v	= components along the nonrolling wind axes

Superscripts

$(\dot{\ })$	= $d/dt(\ )$
$(\ddot{\ })$	= $d^2/dt^2(\ )$
$(\ )'$	= rolling velocity fixed frame

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Introduction

MANEUVERING re-entry of strategic missiles is attractive for the evasion capability it furnishes against target defenses. Three vehicle configurations may be considered for this mission[1]: cruciform, variable-trim bank-to-turn, or fixed-trim. Of these, the simplest design is fixed-trim, wherein the maneuver level capability of the vehicle is not controlled but fixed by body geometry. The vehicle is steered in a bank-to-turn mode, adequate for most purposes, but potentially troublesome in the terminal phase for lack of precision, especially with a severe combination of roll rate limit and roll control system lag.[2] The technical material presented herein deals entirely with the fixed-trim terminal steering case. The objectives are to synthesize a closed-loop steering law that compensates for roll lags while maintaining moderate roll rates.

A simplified model is presented for analyzing the fixed-trim steering problem on a digital computer. The vehicle motion is assumed to be approximated as a point mass acted upon by lift, drag, and gravity forces. The navigation system is idealized as error free, and the roll-autopilot/actuator/airfame is initially assumed to be a first-order system. An extension of the first-order control system to include missile and actuator dynamics is made later in the analysis. The numerical model also serves as a departure point for analytical investigations of the steering problem.

For analytical work, the equations of motion for the vehicle are first rewritten in a nonrolling velocity-fixed system. Using this system of equations, one may derive a steering law that aligns the vehicle's velocity vector with its sight line; however, once this has been accomplished, the roll control must "chatter" in order to keep the velocity and sight-line vectors aligned. An extension of this steering law to preclude wild excursion in roll, once the sight line has been reached, is inhibited by the high order and nonlinear features of the system of equations describing the guidance problem.

To circumvent these difficulties in treating the guidance problem analytically, a systematic order reduction of the system of equations is carried out that simplifies a determination of the control law effective in meeting the objectives of this study. The foundation for a reduction in the order of the state system is prepared by performing a transformation of varibles that results in the introduction of the vehicle's lift force orientation into the kinematic description of the motion. When the motion is subsequently linearized about the sight line to the aimpoint, the flight path and heading angles become "ignorable" state variables. Further order reductions in the state system are obtained by assuming constant flight speed and a small autopilot time constant. The final reduced-order system is essentially a polar coordinate description of the motion of the sight line about the velocity vector of the vehicle. By virtue of the simplicity of the reduced system of equations, one is able to deduce a class of feedback control laws for the guidance problem. However, in view of one of the assumptions made in arriving at the reduced-order model, the control law is only effective if the roll control system lag is reduced or compensated.

If one presumes that the missile, autopilot, and actuator contributions to the roll-system lag are unalterable elements of the system, then any reduction in lag must be accomplished through guidance compensation. In this regard, the development of the reduced-order model and the feedforward compensation device[3-5] of classical control theory are used to synthesize a compensation network.

With a feedback control law and compensation technique having been developed, the effectiveness of the guidance algorithm is then examined computationally. Numerical results are presented first for the first-order roll system model to verify the performance of the feedback control law and feedforward compensation technique.

The performance of the guidance algorithm with the first-order roll model having been verified, a comparison of the present steering law with existing guidance laws is made using a roll system model that accounts for missile and actuator dynamics.

Following the comparison, conclusions are made.

Computational Model

To approximate the guidance problem for a fixed-trim re-entry body during terminal guidance, one may 1) neglect the Earth's rotation and curvature and assume constant acceleration due to gravity; 2) neglect pitch and yaw plane dynamics and assume that the angle of attack is small and constant; 3) model the roll-dynamics/roll-autopilot as a first-order system; and 4) ignore errors associated with the navigation and control system. Under these circumstances the equations of motion of the vehicle may be written in the inertial coordinate system of Fig. 1 as

$$\ddot{x}=(D_x+L_x)/m \tag{1}$$

$$\ddot{y}=(D_y+L_y)/m \tag{2}$$

$$\ddot{z}=(D_z+L_z)/m+g \tag{3}$$

$$\tau_\phi\dot{\phi}_v=\phi_c-\phi_v;\ |\dot{\phi}_v|\leq p_L \tag{4}$$

where $q=\rho V^2/2$, $D=qSC_D$, $L=qSC_{L\alpha}\alpha_T$, $V=(\dot{x}^2+\dot{y}^2+\dot{z}^2)^{1/2}$, and the subscripts on the aerodynamic terms mean the projections of the drag and lift forces along the inertial axes. The projections are nonlinear functions of the velocity components and roll angle. By assumption (4), the output vector from the vehicle's navigation system may be regarded as the same as the acceleration, velocity, and position of the re-entry body relative to the target obtained by integrating Eqs. (1-3). In addition, roll angle and roll rate sensors are assumed to provide measurements of these quantities so that the output vector available for guidance implementation is taken to be

$$y=(r,V,A,\phi_v,\dot{\phi}_v)^T \tag{5}$$

All numerical studies conducted herein use systems (1-3) to obtain the vehicle's trajectory and all guidance algorithms develop steering commands in terms of y given by Eq. (5).

Reduced-Order Model

The inertial states of Eqs. (1-3) are not suited for analytical work and the equations may be redeveloped in a velocity-fixed system as:

$$\dot{V}=-D/m \tag{6}$$

$$\dot{\chi}=-L\sin\phi_v/mV\cos\gamma \tag{7}$$

$$\dot{\gamma}=-L\cos\phi_v/mV \tag{8}$$

$$\dot{x}=V\cos\gamma\cos\chi \tag{9}$$

$$\dot{y}=V\cos\gamma\sin\chi \tag{10}$$

$$\dot{z}=-V\sin\gamma \tag{11}$$

$$\tau_\phi\dot{\phi}_v=\phi_c-\phi_v;\ |\dot{\phi}_v|\leq p_L \tag{12}$$

where, for terminal guidance analysis purposes, the acceleration due to gravity has been neglected in comparison with the maneuver level generated by the lift force.

The reader may note that Eq. (6-12) represent a seventh-order system description of the motion. If only motion in the vertical plane is considered, one may pose the problem of aligning the vehicle's velocity vector with its line-of-sight to the aimpoint in minimum time. This problem may be cast in the form of the classical one-dimensional, time-optimal regulator.[6-8]

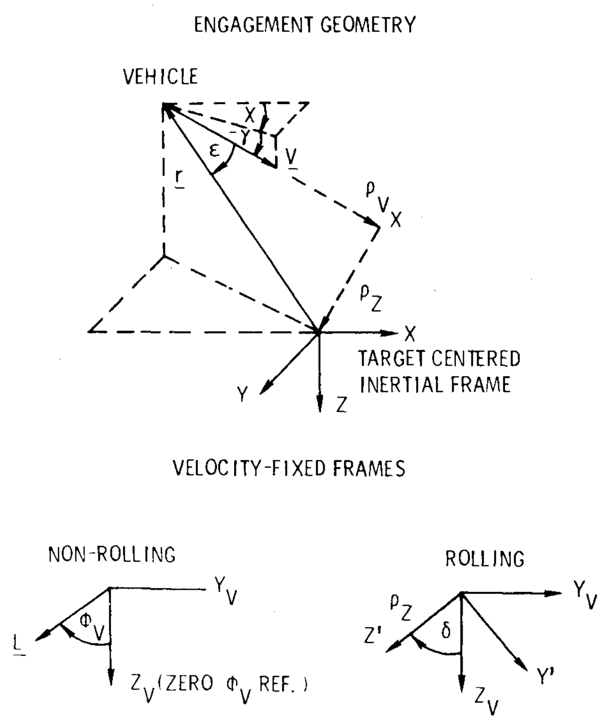

Fig. 1 Engagement geometry and coordinate system descriptions.

The solution of this guidance problem is in the form of an existing guidance algorithm,[1] which simply requires that the vehicle orient its lift vector toward the instantaneous line-of-sight. Referring to Fig. 1, the reader may notice that the desired lift vector will be aligned with ρ_z, the sight-line "error" as viewed in a velocity-fixed reference frame. The direction of the lift will be such that it reduces the "error angle," ϵ, between the velocity and sight-line vectors. Once ϵ has been nulled, however, the time-optimal solution requires that the roll-control "chatter"[9] or instantaneously redirect the lift through 180 deg in order to maintain alignment of the velocity and sight-line vectors. Thus the time-optimal linear regulator has practical value for engagements that are marginal with respect to the sight line being contained within the reachable set. In these cases, the sight line will be approached as quickly as possible. In engagements of primary interest, those for which the sight line may be reached, a more satisfactory control is sought so that once the sight line has been attained, no wild roll gyrations are requried to stay on it.

The combined difficulties of nonlinearity and high system order, presented by Eqs. (6-12), inhibit analytical and/or intuitive approaches to a solution of the fixed-trim steering problem. However, the engagement geometry of Fig. 1 does suggest that a necessary condition for obtaining small aim-point dispersion is that ϵ be kept small. An alternate description of the motion of the vehicle, which involves ϵ, may be obtained by rewriting the governing equation in a velocity-fixed reference frame that rolls with the plane containing ϵ. In Fig. 1, the geometric relationships of the lift vector L, and the sight-line error ρ_z, are sketched as viewed from the rear of the vehicle and along its velocity vector. In the nonrolling velocity-fixed frame, L and ρ_z have roll rates $\dot{\phi}_v$ and $\dot{\delta}$, respectively, and it is difficult to determine the relationship between ϕ_v and δ that is required to reduce ρ_z when the vehicle is spiralling. As viewed from the rolling, or primed, frame, however, ρ_z has no roll rate and is only permitted to translate along the z' axis. The critical motion for the guidance problem can thus be simplified by examining the planar motion of the sight-line error as determined by the component of lift in the plane containing ρ_z.

The detailed development of the motion in the rolling velocity-fixed frame[7] results in the following system of equations. (The essential feature of the transformation is a change of variables describing the kinematics for the problem, a brief derivation of which is given in the Appendix.)

$$\dot{V} = -D/m \tag{13}$$

$$\dot{\gamma} = -(L/mV)\cos\phi_v \tag{14}$$

$$\dot{\chi} = -(L/mV\cos\gamma)\sin\phi_v \tag{15}$$

$$\dot{\rho} = -V\cos\epsilon \tag{16}$$

$$\dot{\epsilon} = (V/\rho)\sin\epsilon - (L/mV)\cos(\phi_v - \delta) \tag{17}$$

$$\dot{\delta} = -(L/mV)\,[\cot\epsilon\sin(\phi_v - \delta) + \tan\gamma\sin\phi_v] \tag{18}$$

$$\tau_\phi \dot{\phi}_v = \phi_c - \phi_v;\ |\dot{\phi}_v| \le p_L \tag{19}$$

where, in terms of the output vector, $y = (r, V, A, \phi_v, \dot{\phi}_v)^T$,

$$\rho = (x^2 + y^2 + z)^{1/2} \tag{20}$$

$$\epsilon = \tan^{-1}(\rho_z/\rho_{v_x}) \tag{21}$$

$$\delta = \tan^{-1}(-\rho_{v_y}/\rho_{v_z}) \tag{22}$$

$$(\rho_{v_x}, \rho_{v_y}, \rho_{v_z})^T = -T(x,y,z)^T \tag{23}$$

$$\rho_z = (\rho_{v_y}^2 + \rho_{v_z}^2)^{1/2} \tag{24}$$

and T is the transformation matrix from the inertial system to the nonrolling velocity-fixed system and is a function of the inertial velocity components.

The transformed system may be approximated for small ϵ and γ as follows:

$$\dot{V} = -D/m \tag{25}$$

$$\dot{\gamma} = -(L/mV)\cos\phi_v \tag{26}$$

$$\dot{\chi} = -(L/mV)\sin\psi_v \tag{27}$$

$$\dot{\rho} = -V \tag{28}$$

$$\dot{\epsilon} = (V/\rho)\epsilon - (L/mV)\cos(\phi_v - \delta) \tag{29}$$

$$\epsilon\dot{\delta} = -(L/mV)\sin(\phi_v - \delta) \tag{30}$$

$$\tau_\phi \dot{\phi}_v = (\phi_c - \phi_v);\ |\dot{\phi}_v| \le p_L \tag{31}$$

The path angles, γ and χ, do not appear on the right-hand sides of Eqs. (25-31) and are thus "ignorable" in the sense that they are not essential to describe the approximate motion of the vehicle. Thus, system (25-31) is reduced to a fifth-order system by ignoring Eqs. (26) and (27). Over several guidance update intervals the speed of the vehicle may be approximated as a constant, V_0. This eliminates Eqs. (25) from the system and allows Eq. (28) to be integrated to yield

$$\rho/V_0 = t_f - t \tag{32}$$

where t_f is the final time and ρ/V the "time-to-go." When Eq. (32) is substituted into Eqs. (29) and (30), the system order is again reduced by two:

$$\dot{\epsilon} = \epsilon/(t_f - t) - (L/mV_0)\cos(\phi_v - \delta) \tag{33}$$

$$\epsilon\dot{\delta} = -(L/mV_0)\sin(\phi_v - \delta) \tag{34}$$

$$\tau_\phi \dot{\phi}_v = \phi_c - \phi_v;\ |\dot{\phi}_v| \le p_L \tag{35}$$

A final order reduction, which is in the singular perturbation category, may be made by removing the roll rate restriction and permitting the roll system time constant, τ_ϕ, to tend toward zero. The roll response is thus instantaneous so that $\phi_v = \phi_c$. When this result is substituted into Eqs. (33-35), the behavior of the vehicle's motion, which was originally described by a seventh-order system, has been approximated by the following system, described by only two first-order differential equations:

$$\dot{\epsilon} = \epsilon/(t_f - t) - (L/mV_0)\cos(\phi_c - \delta) \tag{36}$$

$$\epsilon\dot{\delta} = -(L/mV_0)\sin(\phi_c - \delta) \tag{37}$$

While the system described by Eqs. (36) and (37) is both nonlinear and time-varying, it is now of low order. In addition, the desired roll orientation of the lift vector ϕ_c and the polar orientation δ of the component of the line-of-sight vector, ρ_z, perpendicular to the velocity vector, are related as arguments of the trigonometric terms. This relationship suggests the following interpretation of the fixed-trim guidance problem: the component of maneuver level, $(L/m)\cos(\phi_c - \delta)$, parallel to ρ_z controls the error angle, ϵ, and hence the position error itself, while the component of maneuver level, $(L/m)\sin(\phi_c - \delta)$, controls the rate of rotation, $\dot{\delta}$, of ρ_z about the velocity vector.

The low order and simplicity of Eqs. (36) and (37) may now be exploited in synthesizing a guidance law, which will control the magnitude of the position error, ρ_z, while maintaining moderate rates of rotation of the error about the velocity vector.

Guidance Synthesis

Feedback Control

As mentioned previously, one method to insure small aimpoint dispersion is to control ϵ so that its value is small near the aimpoint. From Eq. (36) one can attempt to control ϵ so that it is nonpositive by requiring that

$$\cos(\phi_c-\delta)=k\epsilon/[(L/mV_0)(t_f-t)] \tag{38}$$

where k is a suitable chosen guidance parameter or gain. To determine acceptable values for the guidance parameter, Eq. (38) is substituted into Eq. (36) to obtain

$$\dot{\epsilon}=\epsilon(1-k)/(t_f-t) \tag{39}$$

which has the solution

$$\epsilon=\epsilon_0(1-t/t_f)^{k-1} \tag{40}$$

Thus for $k\geq 1$, ϵ is either constant or monotonically decreasing, and moreover, when k is strictly greater than one,

$$\lim_{t\to t_f} \epsilon=0 \tag{41}$$

insuring negligible aimpoint dispersion.

One notes that the control law can only be implemented under the mathematical restriction that the right-hand side of Eq. (38) be less than or equal to unity. Because of the physical limitation of fixed-trim vehicles, the maneuver level L/m cannot be adjusted to satisfy the mathematical constraint. Hence, this restriction leads to the requirement that

$$\cos(\phi_c-\delta)=1 \quad \text{for} \quad \epsilon\geq(L/mV_0k)(t_f-t) \tag{42}$$

Combining Eqs. (38) and (42), one obtains the feedback control as a nonlinear combination of the reduced system states with time-varying coefficients:

$$\phi_c=\begin{cases}\delta+\cos^{-1}(\hat{k}\epsilon);\epsilon<1/\hat{k}\\ \delta;\ \epsilon\geq 1/\hat{k}\end{cases} \tag{43}$$

where

$$\hat{k}=k/[(L/mV_0)(t_f-t)] \tag{44}$$

At each guidance update, the nondimensional gain, $\hat{k}$, determines which mode of control should be used. For $\epsilon\geq 1/\hat{k}$,the lift vector is directed toward the sight-line error to produce the maximum rate of reduction in error angle. For $\epsilon<1/\hat{k}$, the lift vector is rotated so that a portion of lift is directed toward the sight line and a portion is directed laterally so that the vehicle will begin circular motion. Therefore, once the sight-line error has been sufficiently reduced, the vehicle will spiral into the target.

Equation (43) represents a class of guidance laws, characterized by the guidance gain, k, that will accurately steer the vehicle. The choice of suitable gains for a given application will ultimately depend upon roll rate restrictions imposed on a given vehicle's control system. As a caveat to the results of this section, the reader must be reminded that an instantaneous control system has been assumed which implies both zero roll response lag and infinite roll rate capability.

Feedforward Compensation

As noted, when the lateral-control-system response is not instantaneous, Eq. (35) must be included in the system of state equations, and the error angle response given by Eq. (40) is no longer valid. If, however, the roll angle lag were small, Eq. (40) could be expected to approximate the error angle response. To examine the effect of small roll angle lags on the roll system response, one observes that, under the assumed first-order autopilot model, Eq. (35), small roll errors, $\phi_e=\phi_c-\phi_v$, result in near-zero roll rates. However, this is contrary to the mathematical and physical analysis that led to the feedback control law. Indeed, the reduced-order system resulted from a description of the motion in a coordinate system that rolled at a rate equal to the polar rate of rotation of the line-of-sight error, ρ_z, about the velocity vector. The feedback control law was derived relative to the rolling coordinate system. In order to permit nonzero roll rates while maintaining small roll errors, one must add a commanded roll rate, p_c, to the right-hand side of Eq. (35), i.e.,

$$\tau_\phi\dot{\phi}_v=\phi_c-\phi_v+p_c \tag{45}$$

With this roll model it is apparent that the roll error, $\phi_c-\phi_v$, may be small [thus insuring that ϵ may be approximated by Eq. (40)] while the body is rolling.

To determine the proper roll rate command to implement Eq. (45), one may temporarily isolate the control system from the remainder of the system dynamics, described by Eqs. (33) and (34), by opening the feedback loop. Thus, Eq. (45) represents a simple linear system that may be analyzed by classical control techniques. In the frequency domain, the roll system lag is represented by a stable pole in the complex plane. By cancelling this pole with a zero, the autopilot lag can be eliminated. Figure 2 is a block diagram illustrating how the pole/zero cancellation is implemented in the control system. The dashed lines in the diagram indicate the compensation network for a first-order system.

Mathematically, the feedforward compensation technique translates into a roll rate commnd, so that Eq. (45) may be rewritten as

$$\dot{\phi}_v=(\phi_c-\phi_v)\tau_\phi+\dot{\phi}_c \tag{46}$$

where $\dot{\phi}_c=\mathrm{d}/\mathrm{d}t(\phi_c)$. To apply the feedforward concept to the fixed-trim control problem, one must differentiate the roll angle command. In order to reduce the sensitivity of the control system to the differentiation process, the roll rate command is approximated in terms of the reduced-order states. Even though the approximation will prohibit exact pole/zero cancellation, a sufficient reduction in the roll error, ϕ_e, should be obtained. Thus, one may simply differentiate the feedback control, ϕ_c, given by Eq. (43), to obtain the roll rate command:

$$\dot{\phi}_c=\begin{cases}\dot{\delta}-\hat{k}\epsilon(2-k)/(t_f-t)\sin(\phi_c-\delta);\epsilon<1/\hat{k}\\ \dot{\delta};\ \epsilon\geq 1/\hat{k}\end{cases} \tag{47}$$

The algebraic details leading to Eq. (47) are provided in Ref. 7. Since the reduced-order system is time-varying and nonlinear, the feedforward compensation technique used here

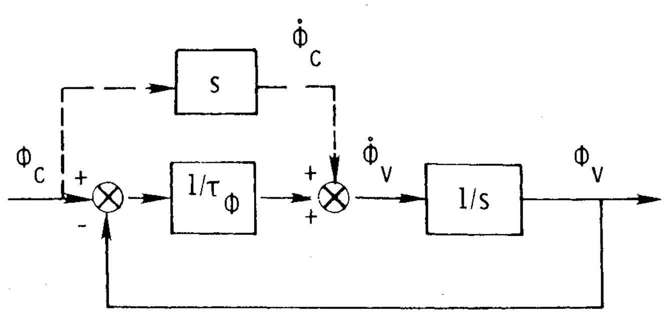

Fig. 2 Block diagram of a first-order roll system illustrating the feedforward compensation network.

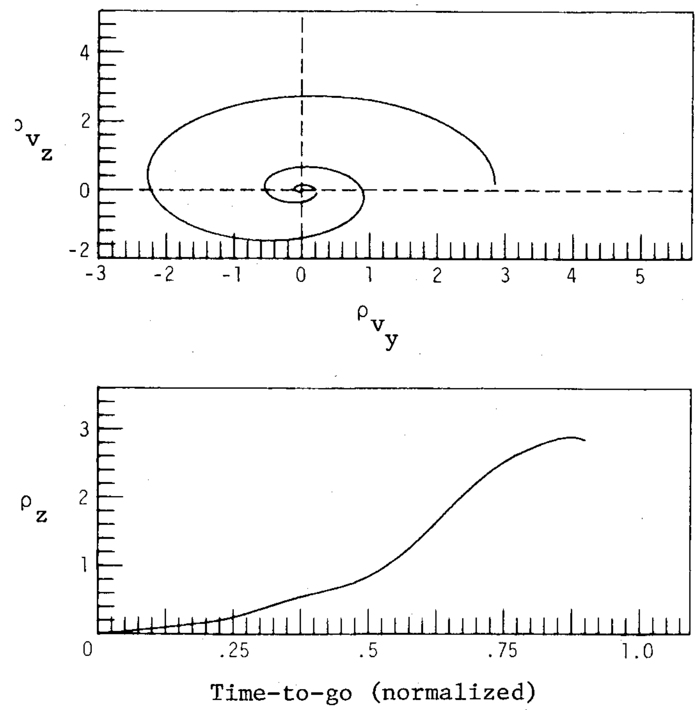

Fig. 3 Normalized line-of-sight error histories for $k=2$.

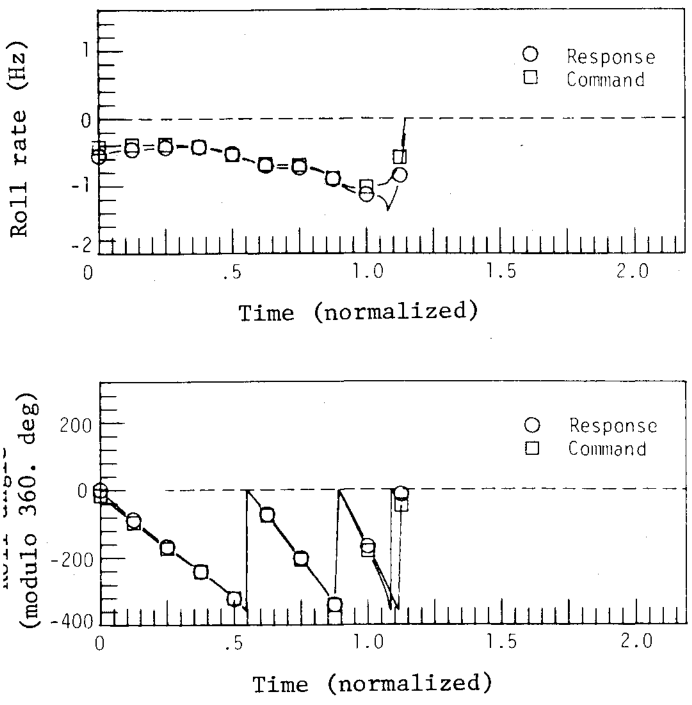

Fig. 4 Roll time-histories for $k=2$.

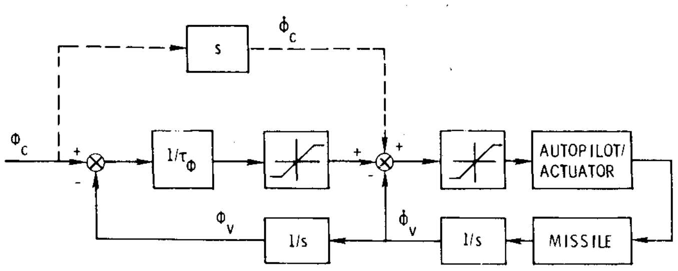

Fig. 5 Block diagram of a fifth-order roll system illustrating the feedforward compensation network.

Table 1 Normalized steering errors at the aimpoint due to initial flight-path angle variation

Initial flight-path angle, γ, deg	Guidance mode: Time-optimal	Dive-line	Present analysis
0	92	132	1
−20	57	88	2
−25	21	123	2
−30	58	100	2
−35	63	149	3
−40	32	185	1
−45	56	18	3
−50	97	129	3

is a generalization of the linear-autonomous system theory concept of pole/zero cancellation.

In summary, Eqs. (43), (46), and (47) represent the guidance scheme synthesized to reduce the error angle between the velocity and line-of-sight vectors according to the time-to-go schedule given by Eq. (40).

Results and Discussion

The computational model, Eqs. (1-4), the guidance algorithm of the present investigation, and two versions of cross-product steering [1,10] were programmed on a digital computer. Numerical studies were conducted to determine the validity of the reduced-order analytical model, on which the present guidance law is based, and to compare the performance of the present law with a representative sample of existing guidance laws. The nominal initial conditions are for a high-supersonic terminal maneuver which, for $\gamma=36$ deg, produces a ballistic (i.e. zero-lift) trajectory passing through the aimpoint.

Present Investigation

Numerical results are presented to examine the performance of the present guidance algorithm. For these studies, a first-order roll system model is used with a time constant of 0.3 s.

The effectiveness of the feedback control law in steering the vehicle is demonstrated in Fig. 3, where the line-of-sight error time histories are presented for $k=2$. In Fig. 4 one observes that the feedforward compensation technique is effective in reducing roll angle errors, $\phi_c-\phi_v$.

Further numerical studies demonstrate that the present steering algorithm is insensitive to initial condition and parameter variations.

Comparison with Existing Guidance Laws

A computational comparison is presented using a roll model that includes missile and actuator dynamics. The implementation of the feedforward compensation network in the higher order roll system is illustrated in Fig. 5. Two versions of cross-product steering are used in this comparison: the "time-optimal" law[1] and a "dive-line" law,[10] which attempts to steer the vehicle along a final prescribed path through the aimpoint. Trajectories were integrated for initial flight-path angle variations from 0 to −50 deg, and the steering errors at the aimpoint are recorded in Table 1. One observes that the time-optimal steering law exhibits somewhat better accuracy than the dive-line law. More dramatic, however, is the order-of-magnitude improvement in the accuracy of the present steering law as compared with existing laws.

Conclusions

A computational model was presented for studying the fixed-trim terminal guidance problem. A time-optimal steering law was described and found to require refinement during the roll modulation portion of the flight. Since the model used to describe the guidance problem obscured a refinement technique, a transformation of the state system was made to illuminate this aspect of the problem.

The transformation of variables and subsequent linearization of the motion resulted in a reduction in the order of the state system for the guidance problem. By inspecting the simple, reduced-order model, a class of steering laws were derived. Large roll angle response lags, characteristic of a bank-to-turn steering mode, were substantially reduced using a feedforward compensation technique.

Numerical studies, using a first-order roll model, indicated that the present guidance algorithm was able to accurately steer the vehicle while maintaining small roll angle errors. Moderate roll rates can be obtained by proper gain selection. A computational comparison with existing steering laws indicated that an order-of-magnitude improvement in steering accuracy can be expected with the present guidance law.

Appendix

From relations (20-24), one observes that the sight-line vector presented in the rolling velocity-fixed frame may be written as:

$$\boldsymbol{\rho} = (\rho\cos\epsilon, o, \rho\sin\epsilon)^T \tag{A1}$$

The time derivative of $\boldsymbol{\rho}$ may be written as

$$\dot{\boldsymbol{\rho}} + \boldsymbol{\omega} \times \boldsymbol{\rho} = -\boldsymbol{V} \tag{A2}$$

where

$$\boldsymbol{\omega} = (\dot{\delta} - \dot{\chi}\sin\gamma, \dot{\gamma}\cos\delta + \dot{\chi}\sin\delta\cos\gamma, -\dot{\gamma}\sin\delta + \dot{\chi}\cos\delta\cos\gamma)^T \tag{A3}$$

is the angular velocity of the rolling frame, and

$$\boldsymbol{V} = (V, o, o)^T \tag{A4}$$

is the velocity of the missile presented in the rolling frame.

By substituting Eqs. (A1), (A3), and (A4) into Eq. (A2), and expanding Eq. (A2) into components, one obtains

$$\dot{\rho}\cos\epsilon - \rho\dot{\epsilon}\sin\epsilon + \rho\sin\epsilon(\dot{\gamma}\cos\delta + \dot{\chi}\sin\delta\cos\gamma) = -V \tag{A5}$$

$$\rho\cos\epsilon(-\dot{\gamma}\sin\delta + \dot{\chi}\cos\delta\cos\gamma) - \rho\sin\epsilon(\dot{\delta} - \dot{\chi}\sin\gamma) = o \tag{A6}$$

$$\dot{\rho}\sin\epsilon + \rho\dot{\epsilon}\cos\epsilon - \rho\cos\epsilon(\dot{\gamma}\cos\delta + \dot{\chi}\sin\delta\cos\gamma) = o \tag{A7}$$

If $\dot{\gamma}$ and $\dot{\chi}$ are replaced with Eqs. (7) and (8), and Eqs. (A5-A7) are solved for $\dot{\rho}$, $\dot{\epsilon}$, and $\dot{\delta}$, one obtains Eqs. (16-18).

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References

[1] Page, J.A. and Rogers, R.O., "Guidance and Control of Maneuvering Re-entry Vehicles," *Proceedings of the 1977 IEEE Conference on Decision and Control*, Vol. 1, New Orleans, La., Dec. 7-9, 1977, pp. 659-663.

[2] Balbirnie, E.C., Sheporaitis, L.P., and Merriam, C.W., "Merging Conventional and Optimal Control Techniques for Practical Missile Terminal Guidance," AIAA Paper 75-1127, presented at AIAA Guidance and Control Conference, Boston, Mass., Aug. 1975.

[3] D'Azzo, J.J. and Houpis, C.H., *Feedback Control System Analysis and Sythesis*, McGraw-Hill, New York, 1966. pp. 529-530.

[4] Graham, R.E., "Linear Servo Theory," *Bell System Technical Journal*, Vol. 25, 1946, pp. 616-651.

[5] Moore, H.R., "Combination Open-Cycle Closed-Cycle Systems," *Proceedings of the Institute of Radio Engineers*, Vol. 29, 1951, pp. 1421-1432.

[6] Bryson, A.E. and Ho, Y.-C., *Applied Optimal Control*, pp. 110-117, John Wiley and Sons, New York, 1975, pp. 110-117.

[7] Gracey, C., "Fixed-Trim Re-entry Guidance Analysis," Ph.D. Dissertation, Virginia Polytechnic Institute and State University, Blacksburg, Va., April 1981.

[8] Leitman, G., *An Introduction to Optimal Control*, McGraw-Hill, New York, 1966, pp. 48-53.

[9] Kelley, H.J., Shinar, J., and Bar-Moshe, D., "Variable-Sweep Optimization," *Automatica*, Vol. 15, 1979, pp. 403-409.

[10] Cameron, J.D.M., "Explicit Guidance Equations for Maneuvering Re-entry Vehicles," *Proceedings of the 1977 IEEE Conference on Decision and Control*, Vol. 1, New Orleans, La., Dec. 1977, pp. 670-678.